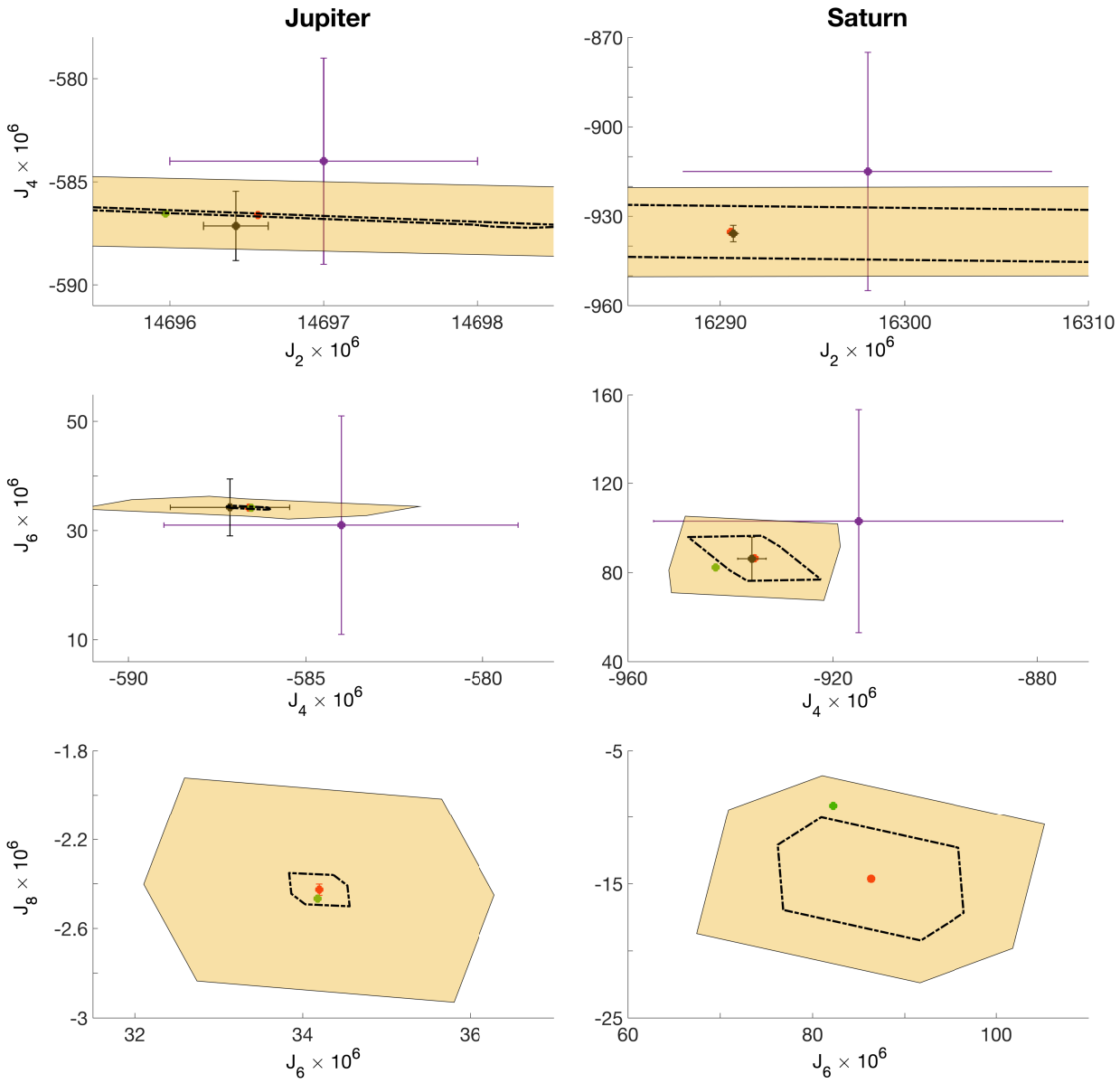

Supplementary Figure 1: Historical (pre Juno and Cassini) measured values of J_2, J_4 and J_6 and their uncertainty for Jupiter (left) and Saturn (right) and the Juno and Cassini measurements (including J_8). Voyager values (purple) are from Campbell and Synnott (1985) for Jupiter and Campbell and Anderson (1989) for Saturn, and Cassini values (brown) are from Jacobson (2003) for Jupiter and Jacobson et al. (2006) for Saturn. Recent Juno and Cassini Grand Finale values (red) are from Iess et al. (2018) for Jupiter and Iess et al. (2019) for Saturn. The effective uncertainty due to dynamics assuming no knowledge on the flow and taking the widest possible range of internal flows (see Kaspi et al., 2017 for details) centered around the Juno/Cassini-measured values appears in yellow, with the contour for $H = 3000$ km (Jupiter) and $H = 10000$ km (Saturn) presented by a dashed line. In green is the difference between the recent Juno/Cassini measurement and the best fit flow profile presented in this paper ($J_n - \Delta J_n$), where ΔJ_n is taken from column 7 in Tables 1 and 2 (ΔJ_2 for Saturn is out of the range presented in the graph), giving our best estimate to what internal models (with no dynamics) should match to.